Warm-up!

For each of the following polynomials, identify the degree and the leading coefficient. Then which way the ends of the graph point.

$f(x) = -4x^5 + 8x^3 + x^2 + 7$

Degree: Leading Coefficient: Left End: Right End:

$g(x) = 16 - x^2$

Degree: Leading Coefficient: Left End: Right End:

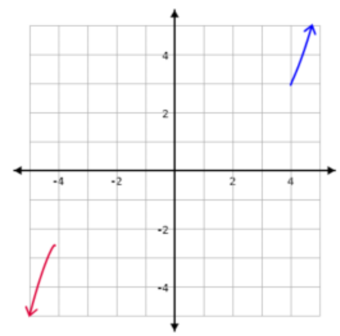
$h(x) = 2x^3 - 11x + 1$

Degree: Leading Coefficient: Left End: Right End:

Unit 1 Day 3 (1.5+1.6)

- Polynomial End Behavior
- Limit Notation
- Determining Polynomial Degree From a Table

Homework: Polynomial Degree and End Behavior



Characteristics of Polynomials

1. Variables have whole exponents

2. Real coefficients

3. No division by variables

Polynomial Vocab

- Degree
- Coefficient
- Leading Term
- Leading Coefficient
- Monomial/Binomial/Trinomial
- Constant/Linear/Quadratic/Cubic/Quartic
- End Behavior

NAMING POLYNOMIALS

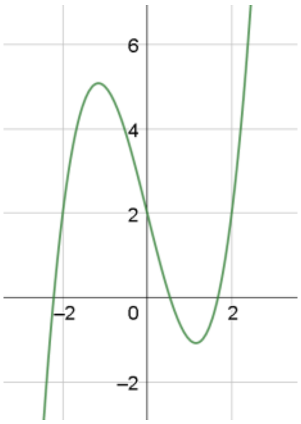
Degree	First Name
0	Constant
1	Linear
2	Quadratic
3	Cubic
4	Quartic
5	Quintic
$n \geq 6$	nth Degree

Terms	Last Name
1	Monomial
2	Binomial
3	Trinomial
$n \geq 4$	Polynomial

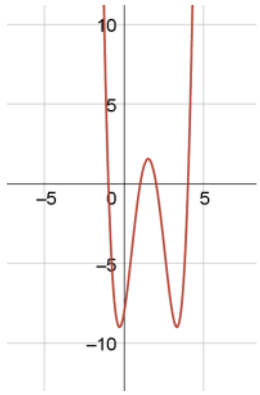
$$f(x) = -2x^3 + 5x^4 - 11x + 7$$

End Behavior

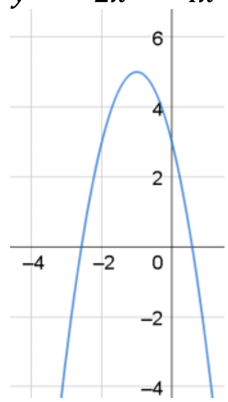
$y = x^3 - 4x + 2$



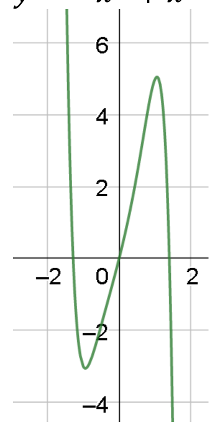
$y = x^4 - 6x^3 + 7x^2 + 6x - 8$



$y = -2x^2 - 4x + 3$



$y = -x^7 + x^3 + x^2 + 4x$



Right end of the graph points up when:

Right end of the graph points down when:

Left end of the graph matches the right when:

Left end of the graph does not match the right when:

How We Write End Behavior: Limits

(Note: Limits are about the journey, not the destination)

$$f(x) = -2x^4 + 10x^2 + 11x - 7$$

$\lim_{x \to -\infty} f(x) =$

x	y

$\lim_{x \to \infty} f(x) =$

x	y

How We Write End Behavior: Limits

$\lim_{x \to -\infty} f(x) = -\infty$ means:

$\lim_{x \to -\infty} f(x) = \infty$ means:

$\lim_{x \to \infty} f(x) = -\infty$ means:

$\lim_{x \to \infty} f(x) = \infty$ means:

Example 1: Determine the end behavior of each polynomial. Express using limit notation.

A) $f(x) = 5x^4 - 11x^3 + 9x + 1$

B) $g(x) = 10 - 12x^3$

Example 2: Determine the end behavior of each polynomial. Express using limit notation.

C) $h(x) = -10x^5 + 14x^4 + x^2$

D) $p(x) = 8x + 10x^4 - 10x^3$

Sometimes we say it with words

$\lim_{x \to -\infty} f(x) = -\infty \Leftrightarrow$ "As input values of $f(x)$ decrease without bound, the output values decrease without bound"

$\lim_{x \to -\infty} f(x) = \infty \Leftrightarrow$ "As input values of $f(x)$ decrease without bound, the output values increase without bound"

$\lim_{x \to \infty} f(x) = -\infty \Leftrightarrow$ "As input values of $f(x)$ increase without bound, the output values decrease without bound"

$\lim_{x \to \infty} f(x) = \infty \Leftrightarrow$ "As input values of $f(x)$ increase without bound, the output values increase without bound"

Why it works...

$$\lim_{x\to\infty}\left(5x^2 - 6x + 10x^3 - 11\right)$$

$$\lim_{x\to-\infty}\left(5x^2 - 6x + 10x^3 - 11\right)$$

Example 3: Determine the end behavior of each polynomial. Express using limit notation.

E) $f(x) = (x-2)^5(x+3)^6$

F) $g(x) = 5(1-x)^7$

Example 4a: Below I tried to give four attributes of a polynomial function. Turns out, by putting these together, I've described something impossible. Why is the scenario impossible? It may help to try to sketch the scenario.

- $\lim_{x\to-\infty} f(x) = \infty$
- $\lim_{x\to\infty} f(x) = -\infty$
- f is concave up on the interval $(0,\infty)$
- The absolute maximum of f occurs at $x = 2$

Example 4b: Below I tried to give four attributes of a polynomial function. Turns out, by putting these together, I've described something impossible. Why is the scenario impossible? It may help to try to sketch the scenario.

- The degree of f is even
- f has one relative maximum. It is $f(2) = 5$
- The absolute maximum of f is 5
- The absolute minimum of f is -3

Moral of the Story of The Previous Two Problems

Unless the domain is restricted, polynomials of an even degree have either an absolute maximum or an absolute minimum. They cannot have both simultaneously, and they cannot have neither.

Unless the domain is restricted, polynomials of an odd degree have no absolute extrema.

Example 5a: Determining Polynomial Degree from Tables

Could the following be points from a linear polynomial? How do you know?

x	-6	-3	0	3	6	9	12	15	18
$f(x)$	21	16	11	6	1	-4	-9	-14	-19

Example 5b: Determining Polynomial Degree from Tables

Could the following be points from a linear polynomial? How do you know?

x	-25	-20	-15	-10	-5	0	5	10	15
$f(x)$	-20	-17	-11	-2	10	25	43	64	88

Example 5c: Determining Polynomial Degree from Tables

Could the following be points from a linear polynomial? Could they come from a quadratic polynomial? How do you know?

x	-12	-10	-8	-6	-4	-2	0	2	4
$f(x)$	7	3	-6	-17	-27	-33	-32	-21	3

Determining Polynomial Degree: Moral of the Story

The degree of a polynomial function can be found by examining the successive differences of the output values over equal-interval input values.
An nth degree polynomial has constant n-th differences.

What does this mean?

Example 5d: Predict the degree of the polynomial that generated the following points

x	-10	-5	0	5	10	15	20	25	30
$f(x)$	1	12	19	22	21	16	7	-6	-23

Example 5e: Predict the degree of the polynomial that generated the following points

x	-16	-8	-4	-2	-1	1	2	4	8
$f(x)$	-8	-5	2	11	20	27	30	27	16

Justifying Our Predictions for the Degree of a Polynomial
Based on a Table

1. Comment on both the input values and the output values
    - e.g. "consecutive and equally spaced input values"
    - e.g. "output values have a constant second difference"
2. There is a difference between saying "the second differences are constant" and "the second differences are increasing by a constant amount."
3. Mention specific values related to the table. It should be clear that you looked at the table and didn't just write a "here's how you justify" sentence you memorized ahead of time. You can write your perfect sentence, but then add in proof you looked at the table.
4. On any assessment this semester, this justification will be either fill in the blank or multiple choice. Second semester you will have to write the sentence without any crutch.

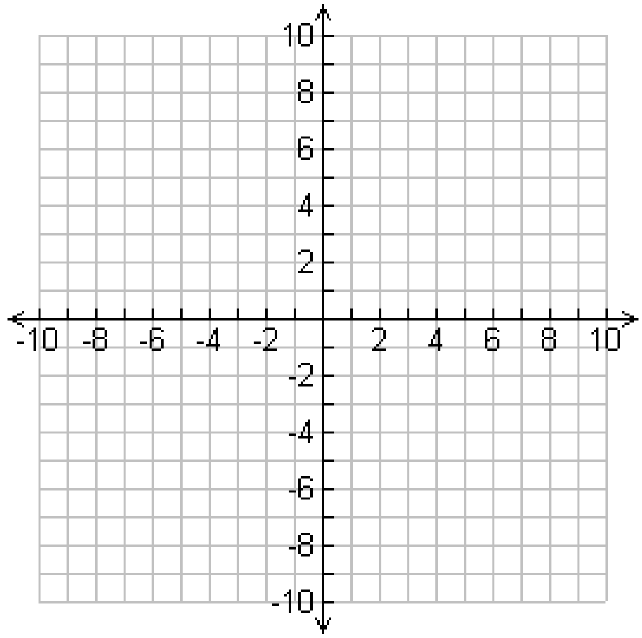
Example 5f: Predict the degree of the polynomial that generated the following points

x	-16	-14	-12	-10	-8	-6	-4	-2	0
$f(x)$	-8	-5	2	11	20	27	30	27	16

Example 6: Sketching

Sketch a continuous function $f(x)$ with the following properties:

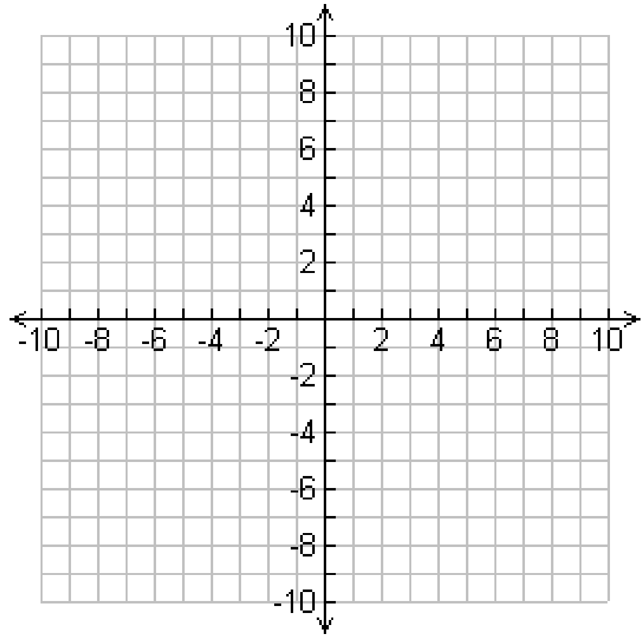
- The rate of change of $f(x)$ is positive on the intervals $(0,2)$ and $(4,\infty)$
- $f(2) = 3$
- $f(x)$ has two relative minima
- $f(x)$ has one local maximum
- $\lim_{x\to\infty} f(x) = \infty$
- $\lim_{x\to-\infty} f(x) = \infty$
- $f(x)$ decreases at an increasing rate on the interval $(-\infty, 0)$



Example 7: Sketching

Sketch a continuous function $f(x)$ with the following properties

- As input values increase without bound, the output of $f(x)$ decreases without bound
- As input values decrease without bound, the output of $f(x)$ increases without bound
- $f(x)$ is decreasing with a decreasing rate of change on the interval $(-6, -2)$
- $f(-6) = 5$ is a point of inflection
- $x = -2$ is the only zero of $f(x)$
- The rate of change of $f(x)$ is -1 at every x-value in the interval $(0, \infty)$
- $f(x)$ does not have an absolute maximum



Example 8: Sketching

Sketch a continuous function $f(x)$ with the following properties:

- Decreasing over the interval $(-\infty, -1]$
- $x = 1$ is a zero of $f(x)$
- The average rate of change of $f(x)$ over the interval $[-3, -1]$ is -2
- The rate of change of $f(x)$ is decreasing on the interval $(-\infty, -3)$
- $\lim_{x \to \infty} f(x) = \infty$
- $\lim_{x \to -\infty} f(x) = \infty$
- $f(-1)$ is a relative minimum value.